The Straw Man Strikes Back:

When Gödel's Theorem is Misused

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There can be no doubt that most philosophy since Descartes has attempted to emulate the success of natural science and mathematics, though there have also been movements, such as romanticism or existentialism, that tried to stem this tide. Given various developments in the hard sciences in the twentieth century, emergent postmodernism found itself straddling a thin fence in this regard. In common with earlier movements, postmodernism extended its hermeneutic of suspicion to science along with other supposedly dogmatic forms of knowledge. However, postmodern writers have also had the luxury of conscripting certain conclusions of science and mathematics to support their cause, thus seeking to use formal knowledge to undermine formal knowledge.

There are two such conclusions in particular to which postmodernists frequently appeal in order to show that reason itself collapses under its own weight. One is Heisenberg's uncertainty principle, which states that one can ascertain either an electron's position or its velocity, but never both. Now, one might think that this is not much of an obstacle for most of our knowledge since few of us ever bother about trying to nail down the precise parameters for subatomic particles, but some writers (including Heisenberg himself[1]), have extended this restriction in physics to question all of knowledge.[2]

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A similar scenario has occurred with regard to an even more difficult principle, Kurt Gödel's Incompleteness Theorem, the topic of this paper. We will describe this principle in detail below. For now, let us simply state that it arose in connection with the attempts by Bertrand Russell and Alfred North Whitehead to generate a complete axiomatic system of arithmetic (as described in the nineteenth century by Giuseppe Peano[3]) from pure logic. Gödel showed that this task is not possible. Any such system will contain true statements that cannot be derived from the system.

The property of completeness, along with soundness, is what makes reasoning within any particular system possible. Soundness is the property that a statement within a system has to be consistent with all of the other statements in the system. Completeness demands that all of the statements within a system are subject to the same rules, viz. that each must follow the same laws of inference as all of the others. In other words, any statement within a system is either given as an axiom or can be derived within the system (completeness) and cannot contradict any other statement in the system (soundness).

To clarify these two properties, let us imagine a system in which there is a single axiom, namely that

(1) A figure with n angles has exactly n sides.

We can then infer within this system that

(2) A figure with four angles has exactly four sides.

But we would violate the property of soundness if we concluded that

(3) A figure with four angles has exactly five sides.

And the property of completeness would become a casualty if we stated within that system:

4) A figure with five angles has the shape of the U.S. Department of Defense.

This last statement may be true, but it is not included with the statements that can be accommodated to the system. If any extraneous information can be brought into a system at any time, there would be no point in attempting to derive conclusions by following the inferential rules of the system. An incomplete system makes reasoning within that particular system pointless.

And so Kurt Gödel came along and showed that Russell and Whitehead's attempt to derive Peano's arithmetic from pure logic could never lead to a complete system.[4] His proof was so compelling that Russell and Whitehead immediately dropped their projects. Gödel's theorem (hereafter: *GT*) also undercut the work of Gottlob Frege and David Hilbert, who were on similar quests. On the other hand, Ludwig Wittgenstein, who routinely waved off what he did not understand, dismissed it as a "logical parlor trick."[5]

Let me clarify here that Gödel's theorem (*GT*) is not a paradox. The fact that formal logic can lead to paradoxes has been well known for a long time. A popular book makes it appear as though Russell ceased working on the *Principia Mathematica* (hereafter: *PM*) when he came up against a paradox that he could not resolve;[6] but, in fact, the entire work was conceived with the paradox in mind and a strategy to resolve it.[7] However, Gödel's discovery was of a very different kind. It did not offer a way out, but shut the whole project down without the possibility of appeal.

Now, again, one might react by yawning. Since few of us commit too much time to deriving Peano's arithmetic from logic, this limitation hardly seems to be all that serious. But again, other writers have seen far more serious consequences radiate from *GT*, questioning the very fabric of knowledge. The question is whether these alleged broader implications to *GT* have serious merit.

Let us look at how the postmodern philosopher Jean-Francois Lyotard makes this application. It would be naïve to think that Lyotard would not have embraced his post-modern position if it had not been for Gödel; he used other factors to substantiate his claims as well. Nevertheless, it so happened that Lyotard's understanding of GT played right into his agenda.

First, Lyotard applies Gödel not just to the derivation of arithmetic from logic but to the very system of arithmetic:

> Now Gödel has effectively established the existence in the arithmetic system of a proposition that is neither demonstrable nor refutable within that system; this entails that the arithmetic system fails to satisfy the condition of completeness.[8]

As we shall see below, this is already an overstatement of the impact of GT. But Lyotard is not content to leave it at that. What may have been little ripples from mathematical logic turns into a veritable tsunami breaking forth over all of knowledge.

> Since it is possible to generalize this situation, it must be accepted that all formal systems have internal limitations.[9]

This is a giant leap for humankind. And in Lyotard's view, what applies to all formal systems must then also extend to ordinary language if it is to be based on formal systems.[10]

> This applies to logic: the metalanguage it uses to describe an artificial (axiomatic) language is "natural" or "everyday" language; that language is universal, since all other languages can be translated into it.[11]

Astoundingly, this generalization actually assumes the success of a project even bigger than the Russell-Whitehead project, namely the reduction of all language to logical formalization, something that, given the connotative side of language, could never be brought off. Still, Lyotard carries on with his assessment:

> [Metalanguage] is not consistent with respect to negation–it allows the formation of paradoxes.[12]

Aside from continuing the sweeping generalizations, this sentence seems to demonstrate a confusion between paradoxes and GT. Still, once we've gone this far, it's only a small step for one to declare:

> This necessitates a reformulation of the question of the legitimation of knowledge.[13]

And thus, Lyotard concludes that Gödel's refutation of Russell and Whitehead has contributed to the downfall of all of human knowledge if it is construed in a modern, rationalistic, way.

Now, keep in mind that for Lyotard this is a good thing. Modern human knowledge, with its emphasis on rigid reasoning and quantifiability, has led to the self-destruction of humanity. It is not coincidental for Lyotard that the time of the greatest advances in science and mathematics is also the time of genocide and holocausts. Hugo L. Meynell summarizes Lyotard's perspective in this way:

> The real issue in modernity is an insatiable and inexorable will-to-power imposing itself by way of rational calculation. The horrifying events of the twentieth century, of which the bombing of Hiroshima and the camp at Auschwitz are outstanding examples, have utterly discredited the project of modernism so far as Lyotard is concerned.[14]

In short, mathematical precision was a major driving force in the oppressive and genocidal mindset of modernism. Fortunately, Gödel has supposedly demonstrated the unreliability of quantificational thinking and, thereby, of all modern thinking.

In the rest of this paper, we will show more precisely what Gödel actually did, and how far his theorem can actually be applied. It will not surprise our readers that we believe the postmodern thinkers who have recruited Gödel to fight for their cause have placed their fate in the hands of a straw man. *GT* has extremely limited applicability.

Summary and Purpose of GT

GT (or, more accurately, Gödel's *First* Incompleteness Theorem)[15] states that *a formal system of arithmetic (whose axioms include those of Peano) is either omega-inconsistent or incomplete.*[16] Further additions have generalized this theorem, which, if nothing else, allow us to reduce the above statement to something a little more understandable. Gödel showed that the theorem works for other formal systems in mathematics; in fact, it works for any system in which the natural numbers can be defined.[17] A generalized version states that *mathematics based on a formal system is either inconsistent or incomplete.*

The key to understanding the nature of Gödel's objective lies in the above phrase "*based on a formal system.*" Once we understand what this phrase means, we can realize that Gödel was not at all interested in promoting skepticism concerning logical or mathematical systems *per se*. The crucial point is the adjective "formal" as applied to systems. "Formal" here has a very specific meaning, and it does not refer to regularity of structure. In the argot of the philosophy of mathematics, "formal" means "generated by human beings on the basis of logical inferences."

The contrary to a "formal" understanding of mathematics is a "Platonic" interpretation (with "intuitionism" being halfway in between). In a Platonic view, numbers and logical relationships are real in themselves; the mathematician simply discovers what has always been true, regardless of anyone's awareness. All mathematical and logical knowledge is fixed by the underlying reality. In the "formal" view, on the other hand, mathematics is a construction built upon commonly accepted axioms, whose status is never more than heuristic. There is no objective reality which mathematical conclusions express; the whole task of mathematics is one of derivation, not discovery.

Gödel was a confirmed Platonist.[18] He believed that mathematics and logic could be absolute and certain because they mirrored the true reality of the mathematical world. His objective was to nullify the formalism of Russell, Whitehead, Hilbert, *et. al.* so as to demonstrate the truth of the Platonic view. Consequently, to use *GT* as a means of arousing skepticism concerning the subject matter of mathematics is to look at it backwards from Gödel's perspective. *GT* should lead us to skepticism concerning formal systems and to an appreciation of the finitude of the human mind so that we can accept the Platonic understanding, which, according to Gödel, alone grants certainty.

We can clarify Gödel's intent by looking at it as an example of transcendental methodology, viz. to assume that a given phenomenon is true and certain and then to ask what the necessary conditions are for the phenomenon to be true and certain. In this particular case, the phenomenon in question involves the given fact that mathematical knowledge is certain. Who would doubt the truths of arithmetic or, thereby, the truth of Peano's axioms, which are simply principles underlying arithmetic? Then, given such undisputed certainty, we can ask under what conditions mathematical knowledge can be certain? Gödel's answer is that it cannot be so within the formalist framework because such a system will always remain incomplete. Nor can we rely on intuition because it will always be suspect. However, we can find the requisite certainty in a Platonic framework. Therefore, as we proceed to scrutinize Gödel's theorem, we need to keep in mind that he was, in fact, committed to the completeness of logical and mathematical systems, but he opposed the effort to derive this completeness by the criteria of formalism.

Formal Systems

A formal system deals with a set of symbols that do not have any meaning in themselves. These symbols are then manipulated according to pre-set rules. By putting the symbols in a certain order, one can get a "sentence". Sequences of "sentences" formed according to the rules of inference are steps in a "proof"; all "sentences" in every such sequence are considered proven provided that the sequence starts with a "theorem" (a previously proven "sentence") or an "axiom" (a "sentence" assumed as proven from the start). "Sentences" may be well-formed or ill-formed.

An example may help with the above. We will create an arbitrary system. For the "axiom," we'll use the "sentence" (11 + 1) = 111. Next, we'll allow ourselves two rules of inference. Lower case letters stand for a sequence of zero or more symbols of the same type: thus, x could stand for 1, 11, 111, 11111, and so on, but not 11+11. These rules of inference are:

1) If (x + y) = z is a "theorem", then so is (x1 + y) = z1.

2) If (x + y) = z is a "theorem", then so is (y + x) = z.

Let us start deriving "theorems" starting from our "axiom". Since (11 + 1) = 111 is a "theorem" (as all "axioms" are automatically "theorems"), so is (111 + 1) = 1111 by the first rule of inference. By the second rule and the previous "theorem", (1 + 111) = 1111 is a "theorem" as well. The next few "theorems" would include (11 + 111) = 11111, (111 + 11) = 11111, and so on.

The most important part of this formal system, and the reason for the overabundance of quotation marks above, is that the system has no inherent meaning. One can play around with it, manipulate the symbols according to the rules, and come up with some interesting arrays of symbols, but these symbols do not have any meaning until they are interpreted. This step of interpretation is how one can have a formal system representing logic or arithmetic. For ease of language in this paper we will refer to the meanings of "sentences"; this is really to say "the meaning of the interpretation of the 'sentence' under the standard interpretation." For the same reason we will also drop the use of the quotation marks.

Going back to the above system, one particular interpretation which just so happens to jump out would be that of addition. For example, the sentence (11 + 1) = 111 could be interpreted as "2 + 1 = 3." The first rule of inference could be "if x + y = z, then (x + 1) + y = (z + 1)", with the second as "if x + y = z, then y + x = z." An important thing to note, however, is that we must stay within the system and not let our interpretations run away with us. Even if "1 + 1 = 2" is a true statement, this does not mean that the sentence (1 + 1) = 11 is a theorem in our formal system; we cannot create it with only our axiom and rules of inference.[19]

Gödel's Proof

A closer look at an outline of the proof of this theorem will show what assumptions are necessary in order for the theorem to hold. As we have clarified above, it applies to formal systems, and specifically mathematical formal systems. In order to remain within a mathematical framework, Gödel used a coding by which each symbol in the formal system was identified with a number. Sentences can then be converted into sequences of numbers according to their symbols, and proofs wind up as combinations of the numbers representing the sentences which make up each step of the proof.[20] All of these numbers are unique to the given symbols, sentences, and proofs (the given numbering scheme is irrelevant insofar as the above hold).The formal system can then, in a way, make statements about itself.[21]

Another important aspect about this coding scheme is that it uses recursive functions. Gödel spends a good amount of his paper laying out the precise formulation of various relations which he needs in order to come up with his "provability" relation. By showing that these relations are all recursive and that all recursive relations are definable within the system, Gödel proves that he can a) use his theorem, b) use it independently of any specific interpretations, and c) generalize his theorem.[22]

Once Gödel has defined his code and shown that one can create valid sentences in the formal system which states things (upon interpretation) such as "This sentence has the Gödel number *x*," his next step is to create a self-destructive sentence like "This sentence does not have a proof in the current system." If it is true, then there is no proof of it in the current system; this makes the statement true, however, and so the system is incomplete. If it is false (or, equivalently, its denial is true), then there is a sentence which can be interpreted as "there is a proof of me in the current system", although every given set of sentences will not constitute a proof of the sentence in question. This is what is known as "omega-inconsistency": there is no direct inconsistency of the form "x and not-x," but an indirect one which cannot be detected in a finite number of steps within the system.[23]

The following suppositions are therefore necessary for the proof to show that a given system falls prey to either incompleteness or fatal inconsistency (i.e. inconsistency such that it entails that all the sentences—including contradictions—in the system are true):

1. it only applies to formal systems;

2. it only applies to Gödelizable (i.e. encodable) systems;

3. it needs two truth values: true and false;

4. it needs a finite number of formalizable axiom schemata.

In addition, the following point is germane:

5. The Gödel sentence (viz. the sentence that turns out to be undecidable) has limited applicability.

In the rest of the paper, we will argue for each point individually to show that one cannot apply *GT* to reason by itself. We do not think that it can be used to refer to human reason *per se* at all, certainly not without bringing in a host of additional metaphysical assumptions. In particular, we think that if *GT* were to hold for human reason, one would have to be committed to a Platonist framework, but then, paradoxically, it would no longer matter because then there would be an intrinsic rationality to the universe independent of our thinking.

1. Formal Systems

In order to come up with his inconsistent sentence, Gödel relies on the fact that he is operating within a system that can be formalizable, where every mode of inference can be catalogued and detailed, and where every axiom can be labeled. Every aspect of the system must be able to be recorded and manipulated in symbolic format according to specific, deterministic rules.

The question that comes up then is, can human reason be formalized? Remember that "formalizable" entails the derivation of intrinsically empty symbols from heuristic axioms on the basis of stipulated rules of inference. One cannot assume that the content of our minds is formalizable unless one takes specific metaphysical stances on issues concerning materialism, determinism, and strong AI. While some will not consider this to be a problem, there is no way around the fact that one must adopt a particular metaphysics in order to apply the theorem in such a manner. Therefore, the use of *GT* to characterize human reasoning is not merely a matter of mathematical logic.

Even if one is a determinist regarding human reason, a formal system is a closed system with a complete set of axioms and rules of inference already given. Reason, on the other hand, is an open system which can always take on more data, experiences, etc. from outside itself, creating a potentially endless supply of axioms, rules of inference, and basic symbols—unless one wishes to follow Gödel in stipulating a Platonic framework.

2. Gödelizable Systems

One of the most important parts of Gödel's proof is that one can create a code so that there is a way to talk about the system within the system. In that system, there are definitions for natural numbers and operations using them. As natural numbers are definable and usable within the system, and the symbols are able to be encoded by numbers, one can make statements about the sentences (at least upon interpretation).

Thus, *GT* only applies if one can manage to take elements that the system (upon interpretation) describes and encode the symbols of the system by those elements. There may very well be more such symbols than necessary, but without this step one cannot use Gödel. Are there any grounds to believe that human reason refers to anything in the way that *PM* refers to numbers? Perhaps reason refers to ideas like numbers are referred to by *PM*. Unless such an encoding could be found, human reason is non-Gödelizable.

There are several conditions necessary for any such encoding. One is that one must be able to specify what the system is and what the interpretation is. Furthermore, these two aspects must be distinct. As seen above, (11 + 1) = 111 is separate from the interpretation of "2 + 1 = 3".[24]

The other condition needed for the theorem to apply is an isomorphism between the system and the interpretation: there must be a one-to-one correspondence between symbols in one and objects in the other (or whatever can be substituted for "symbols" and "objects"). Starting from this base, reason can be either the system or the interpretation of some other system. Assuming these conditions, can the necessary isomorphism be constructed?

Now, this is a tricky question because, as we mentioned above, Gödel's own Platonic understanding of mathematics already contains an isomorphism because that is the central content of a Platonic view. However, in that case the system is also complete because it has its own reality. In order to be vulnerable to GT, human reason must constitute a formal system (in the technical sense explained above), and so the question is whether the requisite isomorphism can be *constructed.*

One possible formal system to be the counterpart to reason might be language, liable to some artistic (or in this case, logical) license. If language could be understood to refer to things (empirical objects, abstract objects, pure thought, or anything else), then one could match up these referents with the words that describe them and thus have a new Gödel code. But this possibility only leads us to a form of realism: words which denote abstract concepts must have a real referent, and this is not possible in a formal system which deliberately eschews such a Platonic assumption.

Anyone attempting to construct a formal system based on the use of language itself must reckon with the fact that words can have multiple meanings. Consider the matter of equivocal speech. In the sentence "Cinderella went to the ball," "went to the ball" can refer to attending a gala event or attempting to gain control of the object of a soccer match. Thus, some symbols would have multiple referents which can be accommodated by a Platonic view in which words express thoughts, and thoughts exemplify uniquely real ideas, but in a formal system this is not possible. And if we stipulate that there could be an extremely complex encoding scheme which could take care of all potential equivocations, then either a) we have unintentionally recreated a Platonic universe or b) we have created a problem in the other direction by eliminating the reality that we often use different words to refer to a single object.

The other obvious option for encoding would use brain states as the system and reason again as the interpretation. We could even assume that all brain states would be restricted to such states as would be involved with reason (which could include senses, memory, reasoning faculties themselves, and others as desired). This suggestion seems to require the assumption of physicalism with respect to the nature of persons, at least to an extent, so that, yet again, we would have to go beyond the logic itself to metaphysics. In addition, every brain-state would need to correspond uniquely to a specific thought or piece of reason. However, it has been demonstrated that upon injury the brain can sometimes recover lost functions. If the new brain state (taking into account the damage) can be interpreted as the same thought as the previous, undamaged brain state, then the isomorphism under consideration breaks down.

There are only a finite number of brain states, no matter how large this finitude may be. If they already potentially encode every conceivable piece of knowledge, then Socrates was correct in believing that the slave boy already knew how to double the square,[25] and we are once again committing ourselves to a Platonic view, just where Gödel would like us to be.

One could, of course, simply assume that reason is a formalizable system and thus liable to *GT*. Hofstatder mentions that if *GT* is true of reason, we might not be able to know it, just as *PM* can't decide that it is a formal system within itself.[26] But this is merely an appeal to ignorance. It is certainly possible that this could be the case, *ceteris paribus*, but it shuts the door to actual argumentation. It seems that the only way to conclude that *GT* applies to reason is to buy into a set of presuppositions that amounts to a Platonic view, in which case *GT* will not apply.

3. *Truth Values*

In the formal system which Gödel uses, there are only two truth values: true and false. Alternatively, a sentence may not be "well-formed": it simply does not make sense. Such a sentence does not need to be considered true or false as it is not saying anything. What would happen in a system with a larger array of truth values, where Gödel's sentence could be something other than true, false, and nonsense?

Now, let us shift from a hypothetical ideal knowledge of reality, which can only be bivalent, to the certitude with which we actually cling regarding our various beliefs. We do not hold all of our opinions with a probability of 1; some are more probable than others. I may not be certain that P is true; I may think that there is a .736 chance that it is true (more or less) and a .264 chance that it is false. That is to say, while I accept its truth and not its falsehood, I still am leaving some room that it may be false. All probabilities brought up here are levels of confidence, not the actual truth-values of the beliefs. Most of the time (or even all of the time) we use less precise measures of probability: this belief is more probable than that one, that belief doesn't seem very probable, etc. Some beliefs are simply incomparable with others. For still other beliefs, we have no idea whatsoever what their probability would be. From an epistemological standpoint, we may not always consider a proposition to have the inverse probability of its denial. For example, I may think that there is good evidence supporting both the truth and falsehood of the Riemann Hypothesis[27], leaving it so that I am not sure whether it is true or not and at the same time allowing me to have a satisfactory level of confidence in whatever opinion on which I may settle. Pure deductive reasoning must adhere to the law of the excluded middle and probabilistic variants, but in everyday reasoning we are much looser with the rules.

Is *GT* still applicable when brought up inside a system where Gödel's sentence is regarded as "partly true with a chance of falsity?" It no longer states "This statement cannot be proven" but becomes "This statement may not be able to be proven." One must look at an infinity of cases and beyond instead of "true" and "false." Does the statement carry the same force in reasoning where one can accept a half-way view?

GT (in generalized form) states that the given formal system is either inconsistent or incomplete. One inconsistency entails that all propositions within the system are true, making inconsistency a generally undesirable thing; just because the cat is on the mat shouldn't imply that it isn't. If human reason is complete and susceptible to *GT* in any way that makes a difference, then inconsistency would spell disaster for it in the same way in which it would for a formal system.

Does typical human reasoning count as logically consistent? With the broad array of probability levels to assess truth that we use in ordinary life, it seems that we would not pass this test. As we said above, a person can believe that a proposition and its denial both have good evidence, and thus give them both a strong chance of veridicality, even though the person realizes that one must be true and the other false (again, we are not intending to show that the law of the excluded middle does not hold on an ontological level or within a rigidly defined logical system). Such a state is not intended to be permanent, as the person will most likely try to figure out which proposition really is true and so relieve the tension of the contradiction.[28] But in the meantime there is no point in denying that we are frequently afflicted with ambivalence.

However, if human reason can be inconsistent at times, there is no necessity that it is incomplete on a theoretical level. What is more, human reason does not fall prey to the logical problem of every proposition being true. I can be in a state of contradictive tension by believing that Gödel was right, but accepting the possibility that he was wrong, without thereby inferring that invisible pink bunny rabbits are jumping on my bed. It still remains the case that it may not be possible for reason to be complete and consistent at the same time and thus to know all truths and only truths through reason. Such a contradiction is not as dangerous to reason in general as it is to math and deductive logic.

We can avoid strict inconsistency because not all opinions will be held at the level of "100% true"; we can believe every true proposition more strongly than its negation, even a good deal more strongly, and thus be close enough to knowing the complete truth through reason without worrying about strict inconsistency. In theory, we could cling to every truth but one with a subjective probability of 1 and the one exception with a subjective probability of .99 (and to its falsity at .01), and such a system of reasoning would not be considered consistent in the required sense to be vulnerable to GT.

4. Axioms

4.1 Finite Axiom Schemata

Another condition which is necessary for the proof to work is that the formal system has to have a finite number of axiom schemata. The system may have an infinite number of axioms, but they can be grouped together under a single pattern. An example of an axiom schema is one which Gödel himself uses:

(4) $(x)(b \vee a) \supset (b \vee (x)a)$

where x is any variable, a is any formula, and b is any formula which does not have x as a free variable.[29] For example,

(5) If, for every natural number n, either 7 + 5 = 12 or n + 1 > 0, then either 7 + 5 = 12 or for every natural number n, n + 1 > 0

would be a valid instantiation of the schema. Outside of the formal system, it's hard to see why (4) is a schema rather than a full-blown axiom, but that is because it is easy for us to simply plug in the necessary variables and formulae. There is no way within the system, before stating (4), to tell whether or not a given formula has x as a free variable or not; even if there were, there would still be issues regarding quantification over quantified propositions, entailing that higher-order logic would need to be used, even in places where *PM* only requires first-order logic. Thus, there would need to be a separate axiom for every pertinent pair of formulae, which means an infinite number of such axioms. A schema is much nicer to use.

Does human reason rest on a finite number of axiom schemata? Any possible formalizing would place them within a system. We could hold the belief "All Gödel sentences are true and not provable except by axioms not in the system in which they were stated." Or, perhaps, "All denials of Gödel sentences are true, and there only exist proofs of them outside the system in which they were stated." More complex formulations could be developed: "The first Gödel sentence is true, the second is false, and so on." If we could formalize and schematize this statement then we could build a formal system that fulfills all of the requirements of *GT*. There would then be a new Gödel sentence not covered by the schema. This would render the schema false, but by definition the schema is provable within the system (as all axioms are). Therefore, if such a schema were formalizable, then it would only be so within an inconsistent system. As there is no particular reason to assume that there cannot be at least one of these schemata which is consistent with other truths, it is more likely that they are not formalizable.

Note that these schemata do not assume that one can know what the Gödel sentence is in a given system. Even in mathematics, we know what the Gödel sentence is and what it means only because we are looking on from outside the system. We can interpret the system and see the truth, but it would not necessarily be evident from within the system. However, we could know that if human reason were affected by *GT*, there would be a Gödel sentence somewhere. We could go from there without knowing how to explicitly state the sentence.

Even aside from these dubious reasons, what would constitute an axiom for human knowledge? Every experience which one has, every sense impression, every memory, every demonstration from logic would constitute the axiom schemata of this system. If one looks at the potential types of experience, there would be a large number of experiences, and thus axioms. However, not all of these experiences would be considered legitimate; other beliefs would affect the legitimacy of an experience. In addition, one must decide whether the conflicting belief or the experience should be decisive. This process can go on for many levels and include many interactions; there is an infinity of possibilities. One may be able to argue that a given schema covering the above is generally accurate; but what is necessary for the applicability of *GT* is one which is perfectly accurate and complete. Even if a schema could be given, it is still an open question as to whether it could be completely formalized.

4.2 Stable Systems

GT only applies to one system at a time. If one is working with one set of axioms now and another set later, at each point in time one's reason could be incomplete. It would, however, be incomplete in different ways.

One could simply take the union of the sets of axioms of the different sets and create a new system with this union (which, as we are taking a finite union of finite sets, would be finite). Assuming that it would be consistent, it would be susceptible to *GT* and thus incomplete. The non-provable statements in this new system would then be non-provable in any of the original systems. If it were inconsistent, the person would not need to keep all the parts of old systems around, ditching from the old what would be contradictory with the new.

Even in this case, the incompleteness of the system changes. One cannot simply produce a single Gödel sentence and have it apply for all time; a Gödel sentence for a present system can be added as an axiom, and a Gödel sentence for a future system cannot be used until one reaches that system (as it may only be able to be expressed in that system). So even if human reason is incomplete, there do not need to be statements which are forever outside of its grasp based on *GT*. The important thing is that there is no truth which is intrinsically outside of our grasp. Even if no person could have complete knowledge, every truth is potentially provable as we can develop our "systems" of reasoning in different directions as need be.

5. *Applicability of Gödel's Sentence*

In spite of the above arguments, perhaps there are some who still think that *GT* can be applied to reason. If so, what would it show? That human reason is either inconsistent or incomplete. If it is inconsistent, then why not try to fix the inconsistency? If one thinks that there is no problem with inconsistency, then inconsistency does not seem to have the same sort of problems in human reasoning which it would in math and so the theorem loses its bite.

Assuming the consistency of reason, what does incompleteness show? If human reason is, at any given finite time, incomplete, this is hardly a revolutionary thought. Even if it shows that human reason is theoretically incomplete no matter what, this is no different from stating that there could be things forever outside of our experience. This notion may be something interesting to think about, but many schools of thought across the ages have said things of this sort in much more profound ways. Even assuming that human reason is incomplete (which seems to be a likely situation), how does this entail that it is bankrupt? To say that metaphysics cannot know the mind of God is not to say that metaphysics is worthless and unfruitful, or that we could dispense with it.

But would the incompleteness generated from *GT* even show this much? In the mathematical system in which Gödel proves his theorem, his sentence only says that "This sentence cannot be proven."[30] It is not a deep truth which cannot be proven; it is not some interesting fact which we have been trying to obtain. It is a specially constructed sentence which was created for the sole purpose of being self-referential. It may very well be that *GT* has no practical result in mathematics, even assuming that mathematics could be completely formalized; everything which we wished to have proven can still be proven, just not pathological cases like Gödel's sentence. It simply serves one purpose, which is to demonstrate the impossibility of the formal projects attempted by Russell, Frege, Hilbert, and others, but it does not hinder mathematicians in their work. It points us to something curious about human reason which is important in one particular situation without actually showing that anything we wished to know on the basis of reason cannot be known.[31]

Conclusion

GT has its place in mathematics as the theorem which proves that no formal system can ever be perfectly complete. Within this context it works well; outside of it, it flounders. If a given system is not formalizable, encodable, bivalent, and stable, then it cannot be subject to the incomplete/inconsistent dichotomy as established by Gödel. If human reason is inconsistent, then this need not result in the terrors which it would for deductive logic (though some formulations of inconsistency can still be bad enough; we are not advocating pell-mell irrationality). Likewise, in all probability human reason is indeed incomplete, but this need not ruin any of our philosophical travel plans due to Gödel's formulation of the problem. We may not wish to subscribe to a Platonic understanding of mathematics, as Gödel did, but as Christian philosophers we do believe that God is rational and that his rationality is displayed in the universe he created, including its mathematical features.

Notes

1. Werner Heisenberg, *Physics and Philosophy* (New York: Harper & Row, 1958).
2. So, communications professor Eric Mark Kramer states dogmatically concerning Heisenberg and his associates, "They all agree that classical logic, which prejudices thinking by positing axioms and theorems that restrict the world to an either/or closed systematics must be abandoned or fundamentally 'modified.' It may 'feel good' and dispel uncertainty, but it is false." Ignoring the fact that those who make assertions such as this obviously give themselves exemptions from their own conclusions, we see how they are intentionally using an idea derived from science to call science into question. Eric Mark Kramer, *Modern/Postmodern: Off the Beaten Path of Antimodernism* (Westport, Conn.: Praeger, 1997), p. 176.
3. The point of Peano's axioms is, of course, not just to calculate how much 2 + 2 comes out to. This information is probably still best uncovered by taking two apples, adding two more apples, and then counting the entire collection. (Hint: The answer is 4.) Peano produced five axioms that defined the nature of natural numbers. A quick survey of basic textbooks will reveal that different authors present these axioms in different formats, sometimes using 1 as the starting number and sometimes 0, depending on whether one considers 0 to be a natural number, as Peano did. Let n stand for the number 0 (or 1) and let the subsequent integers be considered n's successors. Then any number m is a successor to n. For example, if we start with 0, the number 1 is the first successor to n, also designated as (n+1).

   1) n is a natural number.

   $(\exists x)x=n$

   2) Each natural number has a successor.

   $(x)(\exists y)y+1=x$

   3) The number 0 (or 1) is not the successor of any natural number.

   $\sim(\exists x)n=(x+1)$

4) If the successor of x is the successor of y, then x equals y.

$(x)(y)((x+1)=(y+1)) \supset x=y$

5) Given the following conditions:

a. n is a member of a set A of natural numbers;

b. If x is a member of set A, then x+1 is also a matter of set A;

then c) set A contains all natural numbers.

4. "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme,"" *I. Monatshefte für Mathematik und Physik* 38 (1931): 173-98. One English translation: *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, trans. by B. Meltzer (Mineola, N.Y. : Dover, 1992). In the past, the phrase, "and related systems," has been used to show that an application of Gödel's theorem beyond the *PM* is possible. In this paper, we will show that being "related" also involves numerous restrictions.

5. Rebecca Goldstein, *Incompleteness: The Proof and Paradox of Kurt Gödel* (New York: Norton, 2005), p. 117.

6. William Dunham, *The Mathematical Universe* (New York: John Wiley & Sons, 1994), pp. 213-223.

7. The Russell paradox begins by recognizing that some sets appear to be members of themselves, e.g., the set of all the ideas in the world is itself an idea, while others are not, e.g. the set of all mothers in the world is not also a mother. Now imagine a set R, composed completely and exclusively of all the sets that are not members of themselves and ask yourself whether R is a member of itself or not. If it is not, then it must be a member of itself, in which case it is. But then, if it is, it cannot be a member of itself, and so it is not. Then, however, it must be a member of itself, and we keep going around the vicious circle. Russell sought to avoid this paradox by proposing the theory of types, which decrees unilaterally that no set can be a member of itself (appearances notwithstanding). Thus, for example, the set of ideas should not be thought of as an idea in the same sense as the ideas that compose the set. Whether we might think of this as an acceptable solution or not, Russell considered it to be adequate. Alfred North Whitehead and Bertrand Russell, *Principia Mathematica* to *56 (London: Cambridge University Press, orig. 1910, 1962), p. 37 *et passim*.

8. Jean-François Lyotard, *The Postmodern Condition: A Report on Knowledge*, trans. by Geoff Bennington and Brian Massumi (Minneapolis: University of Minnesota Press, 1984), pp. 42-43.

9. Ibid, p. 43.

10. Lyotard himself opted out of the idea of an ideal language, of course, and chose to understand language along a line similar to Wittgenstein's language games.

11. Ibid.

12. Ibid.

13. Ibid.

14. Hugo L. Meynell, *Postmodernism and the New Enlightenment* (Washington, D.C.: Catholic University of America Press, 1999), p. 103.

15. In the same paper, Gödel published his second incompleteness theorem (or Proposition XI) as well, which states that were an arithmetical system to prove its own consistency, then it would be inconsistent. The first incompleteness theorem (or Proposition VI) is actually a step towards proving this result.

16. Gödel's original theorem states that, "To every omega-consistent recursive class c of formulae there corresponds recursive class-signs r such that neither v Gen r nor Neg(v Gen r) are in Flg(c), where v is the free variable of r," where "Neg x" is the encoded negation of x, "v Gen r" is the encoded generalization of the variable v in the sentence r, and Flg c is the set of consequences of c." Kurt Gödel, *Sätze*, p. 187; Meltzer translation, p. 57.

17. This consequence is due to the fact that any system which can define the natural numbers must be able to support mathematical induction, and from this all recursive relations can be shown to be decidable. If all recursive relations are decidable, then all of the relations which Gödel defines are creatable in the system. Given this fact, he can prove his theorem. Furthermore, Rosser shortly thereafter showed that the proof of the theorem can be modified so that it proves that the given system is either inconsistent or incomplete. B. Rosser, "Extensions of some theorems of Gödel and Church." *Journal of Symbolic Logic*, 1 (1936), pp. 87-91.

18. Rebecca Goldstein, *Incompleteness: The Proof and Paradox of Kurt Gödel* (New York: Norton, 2005), pp. 44-51.

19. Douglas R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid* (New York: Basic Books, 1979), pp. 33-63, gives a good introduction to formal systems, upon which this example is based.

20. Thus, according to Gödel's encoding on p. 179 of his original paper, the symbol "(" is 11, " ")" is 13, "x" is 17, "~" is 5, and "v" is 7, . Then, the sentence (x v ~x) is further encoded as $2^{11} \times 3^{17} \times 5^{7} \times 7^{5} \times 11^{17} \times 13^{13}$, where consecutive prime numbers represent the position within the sentence and the exponents of these numbers represent the sym-

bol at the position. Proofs can be encoded in a similar fashion, with the Gödel number of the proof being 2 to the power of the number for the first sentence multiplied by 3 to the power of the number for the second sentence, and so on.

21. One can, for example, take the number $2^{11} \times 3^{17} \times 5^{7} \times 7^{5} \times 11^{17} \times 13^{13}$ which is the Gödel number of (x v ~x). This number then has properties such as "being divisible by 2^{11}". This particular property would be a way of stating "this sentence has '(' as its first symbol)'."

22. Gödel, *Sätze*, pp. 179-186; Meltzer translation, pp. 49-56.

23. Rosser's revisions take a slightly different route to establish either incompleteness or inconsistency, though using similar principles. Rosser, "Extensions," p. 89. In particular, Rosser notes an equivalence between the predicate "is provable" and "is provable and for an existing proof there is no other sentence with a smaller Gödel number which would also be a proof." This equivalence is not demonstrable within the system, as it requires an assumption of consistency which cannot be proven; however, when looking from outside of the system and assuming consistency, it is obvious. Rosser then proceeds to use this beefed-up provability predicate to produce his undecidable sentence. We might also mention that similar conclusions were reached with different methods by others, most notably perhaps, by Alan Turing whose "machine" was a forerunner of the modern computer.

24. Gödel's theorem works because the seemingly paradoxical statement is a statement concerning what an interpretation of the system says about the system, not a statement of what the system says about itself or what the interpretation says about itself. "This sentence is false" is a paradox and can be dismissed as meaning nothing; Gödel's theorem cannot be dismissed so handily as there are no directly self-referential statements. This is what allows it to transcend the safeguards erected by Russell and Whitehead in *PM*, where the theory of ramified types was intended to avoid these types kinds of referential problems.

25. Plato. *Meno*, trans. by Benjamin Jowett. (http://www.gutenberg.org/dirs/etext99/1meno10.txt, 1999)

26. Hofstatder, *Gödel, Escher, Bach*, pp. 559-85.

27. The Riemann Hypothesis, first formulated by Bernhard Riemann in 1859, is an important unsolved problem in mathematics. For more on it, see E. C. Titchmarsh, *The Theory of the Riemann Zeta-Function*, revised by D. R. Heath-Brown (New York: Oxford University Press, 1986).

28. For more on this, see J. R. Lucas, *Minds, Machines, and Godel*, in Alan Ross Anderson, *Minds and Machines*, (Englewood Cliffs, N. J. : Prentice-Hall, 1964), pp. 43-59

29. Gödel, p. 44.

30. Proposition IX on p. 66 of Gödel's paper shows that there are undecidable propositions besides the one which Gödel constructs in Proposition VI, but by Proposition X the satisfiability of such propositions is equivalent to the satisfiability of the earlier one. Also, in footnote 55 on the same page Gödel mentions that "every formula of the restricted predicate calculus . . . is either demonstrable as universally valid or else that a counter-example exists", though such a counter-example cannot always be shown within the formal system.

31. One possible problem that would remain is that if *GT* applies to reason, then other theorems (such as his second incompleteness theorem or Church's theorem) could apply. As these would have to be covered one by one, they will not be dealt with here; however, anything building off of *GT* will only have impact insofar as it can build off the foundation which *GT* gives.